

# Testing for $w < -1$ in the Solar System

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In scalar-tensor theories of gravity, the equation of state of dark energy,  $w$ , can become smaller than  $-1$  without violating any energy condition. The value of  $w$  today is tied to the level of deviations from general relativity which, in turn, is constrained by solar system and pulsars timing experiments. The conditions on these local constraints for  $w$  to be significantly less than  $-1$  are established. It is demonstrated that this requires to consider theories that differ from the Jordan-Fierz-Brans-Dicke theory and that involve either a steep coupling function or a steep potential. It is also shown how a robust measurement of  $w$  could probe scalar-tensor theories.

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Various observations indicate that the expansion of our Universe is presently accelerated [1]. While still debated, this conclusion appears to be more and more robust and, as a consequence, the discussion has now mainly shifted to explaining the cause of this acceleration. The property of the effective equation of state inferred from the observations,  $w$ , is a key issue in this investigation. In particular, showing that  $w \neq -1$  and/or  $dw/dz \neq 0$  ( $z$  being the redshift) would exclude a cosmological constant, probably the most natural candidate and, hence, would have drastic implications for fundamental physics. Recently, various observations have pointed towards the conclusion that  $w < -1$  [2]. Although far from being settled, this would also have important consequences since such an equation of state cannot be achieved by quintessence models [3] for which  $-1 \leq w \leq 1$ . “Phantom models” [4], consisting in a scalar field with a minus sign in front of the kinetic term, are very often advocated in order to explain  $w < -1$ . These models are plagued by various theoretical problems such as, for instance, their stability when interactions with other fields are taken into account.

However, another route can be investigated since theories where the gravity sector is modified, *i.e.* where gravity is no longer described by general relativity (GR), can also entertain  $w < -1$  even if the matter sector is described in a standard fashion. The prototype of such a theory is a scalar-tensor theory of gravity which is both well-defined and well-motivated [5] as they arise as the low energy limit of string theory. It is worth recalling that they are equivalent to theories where the gravitational action is given by an arbitrary function of the Ricci scalar and also encompass the Jordan-Fierz-Brans-Dicke (JFBD) theory as a particular case. As already mentioned, having  $1 + w$  negative is linked to the fact that the gravity sector is modified but such modifications are strongly constrained by solar system and pulsars timing

experiments. Therefore, one can hope to use these local tests to track the real nature of the dark sector. In this letter, we investigate these issues in general scalar-tensor theories.

In the Jordan frame, *i.e.* in the (physical) frame where the experimental data have their usual interpretation, scalar-tensor theories are described by the action [6]

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g} [F(\varphi) R - Z(\varphi) g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 2U(\varphi)] + S_m[\Psi_m, g_{\mu\nu}], \quad (1)$$

which depends on three arbitrary functions  $F(\varphi)$ ,  $Z(\varphi)$  and the potential  $U(\varphi)$ , only two of which are independent.  $G_*$  is a constant, different from the gravitational constant measured in a Cavendish experiment,  $G_{\text{cav}}$ , as will be discussed in more details below. In the following, the matter action,  $S_m[\Psi_m, g_{\mu\nu}]$ , describes a pressure-less perfect fluid.

In the particular case of a Friedmann-Lemaître-Robertson-Walker (FLRW) Universe, choosing  $Z(\varphi) = 1$ , the field equations reduce to

$$3 \left( H^2 + \frac{\mathcal{K}}{a^2} \right) = 8\pi G_* \frac{\rho_m}{F} + \frac{\dot{\varphi}^2}{2F} - 3H \frac{\dot{F}}{F} + \frac{U}{F}, \quad (2)$$

$$- \left( 2\frac{\ddot{a}}{a} + H^2 + \frac{\mathcal{K}}{a^2} \right) = \frac{\dot{\varphi}^2}{2F} + \frac{\ddot{F}}{F} + 2H \frac{\dot{F}}{F} - \frac{U}{F}, \quad (3)$$

with  $H \equiv \dot{a}/a$ ,  $a$  being the scale factor and  $\mathcal{K}$  the curvature of the spatial sections. The energy density of matter scales as  $a^{-3}$ . These equations should be compared to  $3(H^2 + \mathcal{K}/a^2) = 8\pi G_{\text{cav}}(\rho_m + \rho_{\text{DE}})$  and  $-(2\ddot{a}/a + H^2 + \mathcal{K}/a^2) = 8\pi G_{\text{cav}} p_{\text{DE}}$ , since the equation of state of dark energy is experimentally inferred from the expansion history of the Universe by using the standard Friedmann equations of GR. It follows that  $3\Omega_{\text{DE}} w \equiv -1 + \Omega_{\mathcal{K}} + 2q$  where  $q \equiv -a\ddot{a}/(\dot{a})^2$  is the acceleration parameter,  $\Omega_{\mathcal{K}} \equiv -\mathcal{K}/a^2 H^2$  and that the dark energy density parameter is defined as  $\Omega_{\text{DE}}(z) \equiv H^2/H_0^2 - \Omega_m^0(1+z)^3 - \Omega_{\mathcal{K}}^0(1+z)^2$ , where here and in the rest of this letter the subscript “0” denotes the present day value of the corresponding quantity. Clearly, identifying  $\rho_{\text{DE}}$  and  $p_{\text{DE}}$  from Eqs. (2) and (3) leads to the

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conclusion that, in scalar-tensor theories,  $w$  needs not to be positive definite, depending on the choice of  $F$ . This was already stressed by various works [7] and worked out in the particular case of JFBD theories in Refs. [8].

The present values of  $F$  and its derivatives are constrained by the solar system and pulsars experiments. In order to reveal the link between  $w$  and these constraints, it is more convenient to work in the Einstein frame. The Einstein frame metric,  $g_{\mu\nu}^*$  (all Einstein frame quantities will be denoted with a star), is related to the physical metric through the conformal transformation  $g_{\mu\nu}^* = F(\varphi) g_{\mu\nu}$ . Setting  $F(\varphi) = A^{-2}(\varphi_*)$ , the scale factors and cosmic times in both frames are related by  $a = Aa_*$  and  $dt = Adt_*$ , so that the Hubble parameters are linked by  $AH = H_*(1 + \alpha\varphi'_*)$  with  $H_* = d \ln a_*/dt_*$  and where a prime denotes a derivative with respect to the Einstein frame number of  $e$ -folds,  $N_* \equiv \ln(a_*/a_{*0})$ . Deviations from GR are usually described by two parameters,  $\alpha$  and  $\beta$ , which are the first and second field derivative of the coupling function  $A(\varphi_*)$ , namely

$$\alpha \equiv \frac{d \ln A}{d\varphi_*}, \quad \beta \equiv \frac{d\alpha}{d\varphi_*}. \quad (4)$$

In this framework, GR is characterized by  $\alpha = \beta = 0$  while the JFBD theory corresponds to a constant  $\alpha^2 \equiv (2\omega_{\text{BD}} + 3)^{-1}$  and  $\beta = 0$ . As mentioned before, in scalar-tensor theories, the Newton constant obtained in

a Cavendish experiment differs from  $G_*$  and is given by

$$G_{\text{cav}} = G_* A_0^2 (1 + \alpha_0^2). \quad (5)$$

The parameters  $\alpha_0$  and  $\beta_0$  are constrained by various experiments. If we define  $\bar{\gamma} \equiv \gamma^{\text{PPN}} - 1$  and  $\bar{\beta} \equiv \beta^{\text{PPN}} - 1$ , where  $\gamma^{\text{PPN}}$  and  $\beta^{\text{PPN}}$  are the post-Newtonian parameters [9], related to  $\alpha_0$  and  $\beta_0$  by  $\bar{\gamma} = -2\alpha_0^2/(1 + \alpha_0^2)$  and  $2\bar{\beta} = \beta_0\alpha_0^2/(1 + \alpha_0^2)^2$ , then the perihelion shift of Mercury implies  $|2\bar{\gamma} - \bar{\beta}| < 3 \times 10^{-3}$  while the Lunar Laser Ranging experiment sets  $4\bar{\beta} - \bar{\gamma} = -(0.7 \pm 1) \times 10^{-3}$ . On the other hand, a bound on  $\bar{\gamma}$  alone is set from the time delay variation to the Cassini spacecraft near solar conjunction, namely  $\bar{\gamma} = (2.1 \pm 2.3) \times 10^{-5}$ , see Ref. [10] for a review. We conclude that

$$\alpha_0^2 < 10^{-5}, \quad \beta_0 \gtrsim -4.5, \quad (6)$$

where the lower bound on  $\beta_0$  arises from pulsar timing experiments [10]. Note, however, that we cannot consider arbitrarily large values of  $\beta_0$  since then the post-Newtonian approximation scheme would breakdown. In this case, the upper constraints should be reanalyzed. We thus loosely assume that  $\beta_0 \lesssim 100$ .

Working out the Friedmann equations in Einstein frame [6], we obtain that the equation of state is given by

$$3\Omega_{\text{DE}} w = -1 + \Omega_{\mathcal{K}} + (1 + \alpha\varphi'_*) \left[ 3 \frac{A^2}{A_0^2(1 + \alpha_0^2)} - 2 \right] \Omega_{\text{m}} - 2(1 + \alpha\varphi'_*)\Omega_{\text{DE}} + 2 \frac{\alpha\varphi'_*(2 + \alpha\varphi'_*) + \varphi_*'^2}{1 + \alpha\varphi_*'^2} - 2 \frac{\alpha\varphi_*'' + \beta\varphi_*'^2}{(1 + \alpha\varphi_*'^2)^2}, \quad (7)$$

where  $\Omega_{\text{m}} \equiv 8\pi G_{\text{cav}} \rho_{\text{m}}/3H^2$  so that  $\Omega_{\text{DE}} + \Omega_{\text{m}} + \Omega_{\mathcal{K}} = 1$ . In the limit of a minimally coupled scalar field, it reduces to the standard relation  $1 + w = 2\varphi_*'^2/3\Omega_{\text{DE}}$ .

At this point, it is of utmost importance to stress that the value of  $\varphi'_*$  is not free. Indeed, in the Einstein frame, the Friedmann equation reads [6]

$$H_*^2 (3 - \varphi_*'^2) = -3 \frac{\mathcal{K}}{a_*^2} + 8\pi G_* \rho_{\text{m}*} + 2V(\varphi_*), \quad (8)$$

where  $V(\varphi_*) = U/(2F^2)$ , and the positivity of the energy density of matter implies that  $|\varphi'_*| < \sqrt{3}$ , as long as  $\Omega_{\mathcal{K}} \ll \Omega_{\text{m}}$  (in the following, we assume that  $\Omega_{\mathcal{K}} = 0$ ).

The expression (7) for  $w$  is completely general but very intricate and hence not so illuminating. However, taking into account that  $\alpha_0$  has to be small and that  $\varphi'_*$  has to be bounded by  $\sqrt{3}$  and, therefore, that  $\alpha_0\varphi'_{*0}$  has to be small as well, the present day value of the equation of state simplifies considerably and reduces to

$$3\Omega_{\text{DE}}^0 (1 + w_0) \simeq 2(1 - \beta_0)\varphi_{*0}'^2 - 2\alpha_0\varphi_{*0}''. \quad (9)$$

This formula turns out to be our main result. The contribution of  $\beta_0$  arises from the term  $\dot{F}/F$  in the right hand

side of Eq. (3). In fact, Eq. (9) shows that  $w_0 < -1$  is always possible provided  $\beta_0 > 1$  (in this case, even if the slow-roll approximation is satisfied, i.e.  $\varphi_{*0}'' \ll \varphi_{*0}'$ ) and/or  $\alpha\varphi_{*0}''$  positive and large compared to  $\varphi_{*0}'^2$ . Both regimes cannot be reached in the case of JFBD theories (except if the time variation of  $G_{\text{cav}}$  is large, see below) and exist even in the limit  $\alpha_0 \rightarrow 0$  so that all local tests can be satisfied.

The amplitude of  $w_0$  depends on the value of  $\varphi_{*0}'$  and  $\varphi_{*0}''$ . Independently of any dynamics, these two quantities are constrained by the bounds on the time variation of the gravitational constant [11]. Defining

$$\frac{d \ln G_{\text{cav}}}{dt} \equiv \sigma H, \quad \frac{d^2 \ln G_{\text{cav}}}{dt^2} \equiv \xi H^2, \quad (10)$$

the parameter  $\sigma_0$  is bounded by  $|\sigma_0| < 5.86 \times 10^{-2} h^{-1}$ . There is no stringent bounds on  $\xi_0$  but we can estimate that, since  $\sigma_0$  has been “measured” during a period of about 20 years, we have  $|\xi_0 H_0^2| \lesssim |\dot{G}/G|_0/(20 \text{ yr})$ . This implies that  $|\xi_0| \lesssim 5 \times 10^8 h^{-1} \sigma_0 \sim 2.5 \times 10^7 h^{-2}$ . Using Eq. (5), one can then express  $\varphi'_*$  and  $\varphi''_*$  in terms of the

parameters  $\sigma$  and  $\xi$ . As long as  $\beta \neq -(1 + \alpha^2)$ , a case we shall discuss later, one arrives at

$$\varphi'_* = \frac{\sigma}{2\alpha} \left( 1 + \frac{\beta}{1 + \alpha^2} - \frac{\sigma}{2} \right)^{-1}, \quad (11)$$

while the second derivative reads

$$\begin{aligned} \varphi''_* = & \frac{(1 + \alpha\varphi'_*)^2}{2\alpha[1 + \beta/(1 + \alpha^2)]} \xi - \varphi'_* \left\{ \varphi'_* \left[ \frac{\beta}{\alpha} - \alpha + \frac{1}{1 + \alpha^2 + \beta} \left( \frac{d\beta}{d\varphi_*} - \frac{2\alpha\beta}{1 + \alpha^2} \right) \right] \right. \\ & \left. + (1 + \alpha\varphi'_*)^2 \left[ \Omega_{\text{DE}} - \left( 3 \frac{A^2/A_0^2}{1 + \alpha^2} - 2 \right) \frac{\Omega_{\text{m}}}{2} - 1 \right] - \varphi'^2_* \right\}. \end{aligned} \quad (12)$$

Interestingly enough, we see that  $\varphi''_{*0}$ , and hence  $w_0$ , depends on  $\xi_0$  and on the derivative of the  $\beta$  function. The latter is not constrained and we will assume that  $d\beta/d\varphi_* \lesssim \mathcal{O}(100)$  for the same reason as for  $\beta$ . The above formula is rather complicated but, given the previous constraints, we simply have

$$\begin{aligned} \alpha\varphi''_* \simeq & \frac{\xi}{2 + 2\beta/(1 + \alpha^2)} - \frac{\beta}{1 + \beta/(1 + \alpha^2)} \\ & \times \left( 1 + \frac{\beta}{1 + \alpha^2} \frac{1 - \alpha^2}{1 + \alpha^2} \right) \varphi'^2_*. \end{aligned} \quad (13)$$

In particular, the unknown term  $d\beta/d\varphi_*$  does not appear in this approximation.

As mentioned above, the previous considerations are independent from the dynamics. However, from a model building point of view,  $\varphi''_*$  and  $\varphi'_*$  are not independent once the potential  $V(\varphi_*)$  is chosen. They are related through the Klein-Gordon equation which reads

$$\frac{2(X + 1)}{3 - \varphi'^2_*} \varphi''_* + (2 + X)\varphi'_* = -(\alpha X + \alpha_v), \quad (14)$$

with  $X \equiv \Omega_{\text{m}}/\Omega_{\text{v}}$ , where  $\rho_{\text{v}} \equiv 2U/(16\pi G_*)$  and  $\alpha_v \equiv d \ln V/d\varphi_*$ . When the kinetic energy of the scalar field is negligible, we have  $\Omega_{\text{v}} \sim \Omega_{\text{DE}}$ .

We now come back to our master equation (9) and analyze the two regimes where  $1 + w_0$  can become negative and even large (in absolute value). The first regime is the natural extension of quintessence models to scalar-tensor theories [12] and corresponds to the situation in which the field is decelerating, i.e.  $\varphi''_{*0} \ll \varphi'_{*0}$ , so that  $\alpha_0\varphi''_{*0}$  is negligible in Eq. (9). Clearly, this requires  $\beta_0 > 1$ . Such a regime, that cannot be reached in a JFBD model, has the advantage to exhibit an attraction mechanism toward GR [13] so that  $\alpha_0$  can dynamically be made small. Since, in this situation,  $\beta_0$  is not close to  $-1$ , Eq. (11) implies that  $\varphi'_{*0} \sim \sigma_0/[2\alpha_0(1 + \beta_0)]$  and Eq. (13) leads to  $\xi_0 \sim \beta_0\sigma_0^2/[2\alpha_0^2(1 + \beta_0)]$ . On the other hand, from the Klein-Gordon equation, one obtains that  $\varphi'_{*0} \sim -(\alpha_0 X_0 + \alpha_{v0})/(2 + X_0)$  and, in order to fulfill the condition  $\varphi'_{*0} < \sqrt{3}$ , one must have  $|\alpha_{v0}| \lesssim 4.2$ . The two

expressions for  $\varphi'_{*0}$  can be used to infer what  $\beta_0$  is and, if we insert the result in Eq. (9), one gets

$$3\Omega_{\text{DE}}^0(1 + w_0) \simeq \frac{\sigma_0\alpha_{v0}}{(2 + X_0)\alpha_0} < 0, \quad (15)$$

in the most interesting limit where  $\alpha_{v0} \gg \alpha_0 X_0$  since, otherwise,  $|1 + w_0| \sim \mathcal{O}(\alpha_0^2)$ . They are two ways to interpret Eq. (15). Either it gives  $w_0$  in terms of  $\alpha_0$  and  $\alpha_{v0}$ , assuming that  $\sigma_0$  is known. Or it provides, for fixed  $\alpha_0$  and  $\alpha_{v0}$ , the minimum value that  $w_0$  can reach assuming, as is the case now, that only an upper bound on  $\sigma_0$  is available. Fig. 1 depicts the above-mentioned minimum value as a function of  $(\alpha_0, \alpha_{v0})$ .

At this point several remarks are in order. Firstly, it is unclear whether the above situation can be realized without some fine-tuning in a realistic model, where both the coupling function and the potential are running away in order to have attraction towards GR and insensitivity to the initial conditions [14]. Let us illustrate this point on the following particular example. Consider the case where  $\alpha \propto e^{-\lambda\varphi_*}$  so that  $\beta_0 = -\alpha_0\lambda$ . Since we need  $\alpha_0 \ll 1$  and, at the same time,  $\beta_0 > 1$ , this means that  $\lambda \gg 1$  which may be considered as unnatural. Secondly, one can reverse the logics and study what a detection of  $w_0 < -1$  would imply on scalar-tensor theories (assuming, of course, that a slow-rolling  $\varphi_*$  is causing the acceleration of the expansion). Besides the fact that the condition  $\beta_0 > 1$  would drastically improve the current limit on  $\beta_0$  and, in particular, exclude GR, it is also interesting to remark that this would link  $w_0$  to the time variation of the Newton constant through the expressions  $\sigma_0^2 \simeq 6\Omega_{\text{DE}}^0(1 + w_0)(1 + \beta_0)^2\alpha_0^2/(1 - \beta_0)$  and  $\xi_0 \sim 3\Omega_{\text{DE}}^0\beta_0(\beta_0 + 1)(w_0 + 1)/(1 - \beta_0) \lesssim 6 \times 10^2$ , a limit sharper than the bound set by local experiments. Finally, it is interesting to notice that  $w_0 < -1$  can be obtained even if the potential energy is negligible (in GR this would correspond to  $w_0 \sim 1$ ). In this case, one can show that  $\varphi'_{*0} \sim (3\Omega_{\text{DE}}^0)^{1/2}$  and  $\varphi''_{*0} \sim -3\Omega_{\text{m}}^0\varphi'_{*0}/2$  so that  $\alpha_0\varphi''_{*0}$  is indeed negligible in Eq. (9). Then, the equation of state can easily be obtained and reads  $1 + w_0 \sim 2(1 - \beta_0)$ . Again  $\beta_0$  is bounded by the constraint on  $\sigma_0$ ,  $1 + \beta_0 < \sigma_0/(\alpha_0\sqrt{12\Omega_{\text{DE}}^0})$ , so that a large

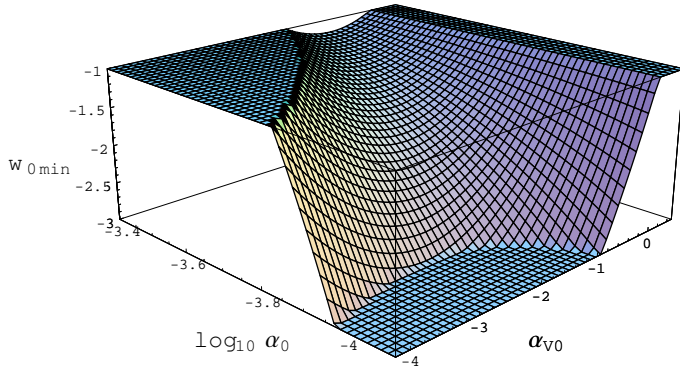


FIG. 1: Minimum value of  $w_0$  as a function of  $(\alpha_0, \alpha_{v0})$  for  $\sigma_0 < 10^{-3}$ .

value of  $w_0$  requires  $\sigma_0 \ll \alpha_0$ .

The second regime corresponds to the case where  $\alpha\varphi''_*$  dominates in Eq. (9). As can be seen in Eq. (13), this is possible if  $\xi_0$  is large and positive and/or if  $\beta_0 \sim -1$ . Strictly speaking, one should in fact consider the limit  $\beta_0 \rightarrow -(1 + \alpha_0^2)$  corresponding, when  $V = 0$ , to the Barker theory [15] in which  $A = \cos \varphi_*$  and  $G_{\text{cav}}$  constant ( $\sigma = \xi = 0$ , whatever the value of  $\varphi'_*$  and  $\varphi''_*$ ). Since  $\alpha = -\tan \varphi_*$ , the cosmological evolution drives the theory away from GR unless a potential keeps  $\varphi_*$  close to 0 until the last  $e$ -folds inducing a large variation of  $\alpha\varphi''_* \sim \mathcal{O}(1)$  in order to have  $1 + w_0 < 0$ . Such a model seems very contrived unless the potential exhibits a slope discontinuity recently.

Let us now assume that  $\xi_0 \sim 0$  since, from dimensional analysis, one expects  $\xi_0 \sim \sigma_0^2 \lesssim 2 \times 10^{-3}$ . In the limit  $\beta_0 \rightarrow -1 - \alpha_0^2$ , Eq. (13) leads to  $\alpha\varphi''_* \sim 2\alpha_0^2\beta_0^2\varphi_{*0}^2/(1 + \beta_0 + \alpha_0^2)$  and  $w_0$  diverges when  $\beta_0 \rightarrow -1 - \alpha_0^2$ . However, the constraint  $\varphi'_* < \sqrt{3}$  implies that

$1 + \beta_0 + \alpha_0^2 \gtrsim \sigma_0/(2\sqrt{3}\alpha_0)$  and, hence, the smallest value that can be obtained is  $(1 + w_0)_{\text{min}} \sim -8\sqrt{3}\alpha_0^3/(\Omega_{\text{DE}}^0 \sigma_0)$ . In particular,  $w_0 \ll -1$  is perfectly possible in this regime if  $\sigma_0 \ll \alpha_0^3$ . Finally, let us see what this regime implies in terms of model building. The Klein-Gordon equation implies that  $2\varphi''_{*0}/3 \sim -\alpha_{v0}/(2 + X_0)$  so that the slope of the potential must be very large,  $\alpha_{v0} \gg \alpha_0^{-1}$ . Moreover since, in this regime,  $\beta_0 < 0$  the theory is driven away from GR and it follows that the potential must be tuned in order to prevent this drift.

As a conclusion, let us summarize our main findings. Firstly, we have confirmed that  $1 + w_0$  is not positive definite in scalar-tensor theories even if all the matter energy conditions are satisfied. Secondly, we have established under which conditions  $1 + w_0$  can become negative given the local constraints coming from solar system and pulsars measurements (in the case of chameleon models [16], it has been argued that  $\alpha_0$  can be of order unity today, a case where large negative values of  $1 + w_0$  can be achieved more easily). We have shown that getting a non negligible deviation from  $-1$  necessarily implies a non vanishing  $\beta_0$  (except if  $\xi_0$  is large), a situation that cannot be reached in the JFBD case. Thirdly, in terms of model building, we have demonstrated that this corresponds either to  $\beta_0 > 1$ , a situation where the scalar field is slow-rolling today and the coupling constant is very steep or to  $\beta_0 \sim -1$  (or  $\xi_0$  large) where the slope of the potential is very large. Fourthly, we have also shown how a measurement of  $w_0 < -1$  could improve the local constraints on the deviations from GR. This highlights the complementarity [17] between cosmological and local tests of gravity. Finally, let us remark that we have only taken into account the local constraints. A next step would be to reconstruct the redshift evolution of the models and to show that they are not pathological [6]. **Acknowledgments:** we thank G. Esposito-Farèse for many enlightening comments and discussions.

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